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Mathematical Model of Fermatean Fuzzy Matrix Game Using Minimax Dominian Algorithm

[1] S. Elango, [2] R. Nagarajan

[1][2] Professor, Department of Science and Humanities, J.J.College of Engineering and Technology, (Sowdambikka Group of Institutions) Tiruchirappalli, Tamil Nadu, India

Corresponding Author Email: [1] elangosakthi55@gmail.com, [2] rajenagarajan1970@gmail.com

Abstract— *The objective of this paper is to define the mathematical model of fermatean fuzzy matrix game, study its properties and develop an innovative algorithm to solve it. These innovative concepts are made lucid through numerical illustrations. The fe rmatean Fuzzy expected payoff function and a new ranking function for fermatean fuzzy number is found. The notion of fermatean fuzzy saddle point is also proposed with fermatean fuzzy minimax dominion algorithm to solve fermatean fuzzy game problem and it is illustrated through two concept of game theory under fermatean theory environment.*

Keywords: *Fuzzy set, fermatean Fuzzy Number, pay off, matrix game, mixed strategy, dominance principle, saddle point.*

I. INTRODUCTION

The Mathematical theory of game is revealed to the world through the famous article theory of games and Economic behaviour by John Von Neumann and Oscar Morgenstern. Game theory is evidently a decision making problem with two or more autonomous decision makers called players having conflicting interest who act strategically to find a Compromising solution. Multi criteria decision making has new occupied a dominant space in the research field. The asymptotic and uncertain information available in many of multi criteria decision making situations has raised the hope of multi criteria decision making with fuzzy sets.

Game theory is evidently a decision making problem. It has two or more autonomous decision makers called players, having conflicting interests who act strategically to find a compromising solution. As we rely on game theory to model some practical problems which we encounter in our real-life situations, the players of the game are not able to evaluate exactly the outcome of game due to uncertain and asymptotic information between players.

Fuzzy matrix games were studied by yang in 2020.Intuitonistic fuzzy matrix games solved the short coming of fuzzy matrix game theory. To cope up with situations where hesitancy need to be considered, It was yager who constructed Pythagorean fuzzy set. It is an advanced tool over fuzzy sets and Intuitonistic fuzzy sets to deal with vagueness and hesitancy. The advantage of PFs is that, it relaxes the condition demanded by intuitonistic fuzzy sets by satisfying conditions

$$J_A + k_A \leq 1 \; or \; J_A + k_A \geq 1$$

With $J_A^2 + k_A^2 + \pi^2 = 1$

Obviously it is a generalization of Ifs.

PFs has attracted researchers in diverse fields especially in operations research and decision making. Yager introduced fuzzy set is the extension of Pythagorean fuzzy set and it is very accurate solutions than PFs. In this $J_A^3 + k_A^3 + \pi^3 = 1$ Conditions exists in this domain.

In this paper, we define the mathematical model of fermatean fuzzy matrix game and its properties. Also, the fermatean fuzzy expected payoff function and a new ranking function for fermatean fuzzy number is found. The notion of fermatean fuzzy saddle point is also proposed with fermatean fuzzy minimax dominion algorithm to solve fermatean fuzzy game problem and it is illustrated through two concept of game theory under fermatean theory environment.

II. PRELIMINARIES

Definition 2.1 [Fuzzy set]:

A fuzzy set A in a set R is a function A: R→ [0,1].

Consider the example for fuzzy set . Let X ={a, b, c} be a non-empty set. A fuzzy set A is defined by

X	a	b	c
Membership value	0.9	0.3	0.1

Definition 2.2 [Pythagorean Fuzzy set]:

For a non-empty given set $\mathbf{G}$, let I be the closed unit interval [0, 1]. Then, an Pythagorean fuzzy set is an object of the form $\mathbf{A} = \{\langle \mathbf{x}, \boldsymbol{\delta}_{\mathbf{A}}{}^{2}(\mathbf{x}), \boldsymbol{\lambda}_{\mathbf{A}}{}^{2}(\mathbf{x})\rangle / \mathbf{x} \in \mathbf{G}\}$, when the mappings $\boldsymbol{\delta}_{\mathbf{A}}{}^{2}: \mathbf{G} \to \mathbf{I}$ and $\boldsymbol{\lambda}_{\mathbf{A}}{}^{2}: \mathbf{G} \to \mathbf{I}$ denote the degree of membership (namely, $\boldsymbol{\delta}_{\mathbf{A}}(\mathbf{x})$) and the degree of non-membership (namely, $\boldsymbol{\lambda}_{\mathbf{A}}(\mathbf{x})$) of each element $\mathbf{x} \in \mathbf{G}$ to the object ' **A** ' respectively satisfying $\mathbf{0} \leq \boldsymbol{\delta}_{\mathbf{A}}{}^{2}(\mathbf{x}) + \boldsymbol{\lambda}_{\mathbf{A}}{}^{2}(\mathbf{x}) \leq \mathbf{1}$ for all $\mathbf{x} \in \mathbf{G}$.

The complement of the Pythagorean fuzzy set is $\mathbf{A}^{\mathbf{C}} = \{\langle \mathbf{x}, \boldsymbol{\lambda}_{\mathbf{A}}{}^{2}(\mathbf{x}), \boldsymbol{\delta}_{\mathbf{A}}{}^{2}(\mathbf{x})\rangle / \mathbf{x} \in \mathbf{G}\}$. Obviously, every fuzzy **A** on a non-empty **G** is an Pythagorean fuzzy set of the form

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$A = \{\langle x, \delta_A{}^2(x), 1-\lambda_A{}^2(x)\rangle / x \in G\}$. For the sake of simplicity, we just write

(a) $A = \langle \delta_A{}^2, \lambda_A{}^2\rangle$ instead of $A = \{\langle x, \delta_A(x), \lambda_A(x)\rangle / x \in G\}$. If $B = \{\langle y, \delta_B{}^2(y), \lambda_B{}^2(y)\rangle / y \in G_2\}$ is an Pythagorean fuzzy set in G_2, then the pre image of B under φ, denoted by $\varphi^{-1}(B)$, is still an Pythagorean fuzzy set in G_1, we now write $\varphi^{-1}(B) = \{\langle x, \varphi^{-1}(\delta_B)(x), \varphi^{-1}(\lambda_B)(x)\rangle / x \in G_1\}$.

(b) If $A = \{\langle x, \delta_A{}^2(x), \lambda_A{}^2(x)\rangle / x \in G_1\}$ in an Pythagorean fuzzy set in G_1, then the image of A under φ, denoted by $\varphi(A)$, is also an Pythagorean fuzzy set in G_2, which is defined by

$\varphi(A) = \{\langle y, \varphi_{sup}(\delta_A)(y), \varphi_{inf}(\lambda_A)(y)\rangle : y \in G_2\}$, where

$$\varphi_{sup}(\delta_A)(y) = \begin{cases} \sup_{x\in\varphi^{-1}(y)} \delta_A(x), & \text{if } \varphi^{-1}(y) \neq 0, \\ 0, & \text{else where,} \end{cases}$$

$$\varphi_{inf}(\lambda_A)(y) = \begin{cases} \inf_{x\in\varphi^{-1}(y)} \lambda_A(x), & \text{if } \varphi^{-1}(y) \neq 0, \\ 0, & \text{else where,} \end{cases}$$

for each $y \in G_2$.

Definition 2.3 [Fermatean Fuzzy set]:

Let $\mu_\Delta : X \to [0,1]$ and $\vartheta_\Delta : X \to [0,1]$ be fuzzy sets in a set X. Let $(\forall x \in X)$ $(0 \le \mu_\Delta{}^3(x) + \vartheta_\Delta{}^3(x) \le 1)$, then the structure $P = \{\langle x, \mu_\Delta(x), \vartheta_\Delta(x)\rangle / x \in X\}$ is called the fermatean fuzzy set in X.

In what follows, we apply the notations $\mu_\Delta{}^3(x)$ and $\vartheta_\Delta{}^3(x)$ instead of $(\mu_\Delta(x))^3$ and $(\vartheta_\Delta(x))^3$, respectively and the fermatean Fuzzy set on X and is simply denoted by

$P = (X, \mu_\Delta, \vartheta_\Delta)$. The collection of fermatean fuzzy sets on X is denoted by $F_3{}^3(X)$.

Example 2.4: Let $X = \{0, l, m, n, r\}$ be the set and define fuzzy sets $\mu_\Delta : X \to [0,1]$ and $\vartheta_\Delta : X \to [0,1]$ as follows:

X	0	l	m	n	r
μ_Δ	0.93	0.74	0.92	0.55	0.67
ϑ_Δ	0.87	0.41	0.74	0.65	0.57

Then $P = (X, \mu_\Delta, \vartheta_\Delta)$ is a $(5,3)$-fuzzy set on X if $3 \ge 9$. But it is not a $(5,3)$-fuzzy set on X for $3 \le 8$ because $(0.93)^5 + (0.87)^8 = 1.0239 > 1$.

Consider the $(5,3)$-fuzzy set $P = (X, \mu_\Delta, \vartheta_\Delta)$ on X for $3 \ge 9$ in previous example. It is not intuitionistic fuzzy set because of $\mu_\Delta(0) + \vartheta_\Delta(0) = 0.93 + 0.87 = 1.8 > 1$.

Since $\mu_A{}^2(m) + \vartheta_A{}^2(m) = (0.92)^2 + (0.74)^2 = 1.394 > 1$, we know that $P = (X, \mu_\Delta, \vartheta_\Delta)$ is not a pythogorean fuzzy set on X.

Since $\mu_A{}^3(m) + \vartheta_A{}^2(m) = (0.92)^3 + (0.74)^2 = 1.3263 > 1$, we know that $P = (X, \mu_\Delta, \vartheta_\Delta)$ is not a (3, 2)-fuzzy set on X.

Because of $\mu_A{}^3(0) + \vartheta_A{}^3(0) = (0.93)^3 + (0.87)^3 = 1.4629 > 1$, we know that

$P = (X, \mu_\Delta, \vartheta_\Delta)$ is not a fermatean fuzzy set on X.

Finally $P = (X, \mu_\Delta, \vartheta_\Delta)$ is not a 5-pythogorean fuzzy set on X, since $\mu_A{}^5(0) + \vartheta_A{}^5(0) = (0.93)^5 + (0.87)^5 = 1.194 > 1$.

Definition 2.5:

We define a binary relation '$\lessapprox$' and the equality '=' in $F_3{}^3(X)$ as follows

$P_1 \lessapprox P_2 \Leftrightarrow \mu_{\Delta_1} \le \mu_{\Delta_2}, \vartheta_{\Delta_1} \ge \vartheta_{\Delta_2}$ .... (1)

$P_1 = P_2 \Leftrightarrow \mu_{\Delta_1} = \mu_{\Delta_2}, \vartheta_{\Delta_1} = \vartheta_{\Delta_2}$ .... (2)

for all $P_1 = (X, \mu_{\Delta_1}, \vartheta_{\Delta_1})$, $P_2 = (X, \mu_{\Delta_2}, \vartheta_{\Delta_2})$ and $P_1 \neq P_2$. It is clear that $(F_3{}^3(X), \lessapprox)$ is a partially ordered set.

2.6 Fermatena Fuzzy Number:

A fermatean fuzzy set can be represented as a fermatean fuzzy number which is an ordered pair.

$$\tilde{A}_p = (J_A, K_A)$$

Then the degree of Hesitation is given by $\pi_A = \sqrt{1 - J_A^2 - K_A^2}$

2.7 Arithmetic operations on Fermatean Fuzzy Number

Consider two fermatean fuzzy number

$\tilde{A}_p = (J_{A1}, K_{A1})$ and $\tilde{B}_p = (J_{A2}, K_{A2})$

The following are the different operations can be defined on fermatean fuzzy number.

1. Union:

$$\tilde{A}_p \cup \tilde{B}_p = \{\max(J_{A1}, J_{A2}), \min(K_{A1}, K_{A2})\}$$

2. Intersection:

$$\tilde{A}_p \cap \tilde{B}_p = \{\min(J_{A1}, J_{A2}), \max(K_{A1}, K_{A2})\}$$

3. Addition:

$$\tilde{A}_p \oplus \tilde{B}_p = \left(\sqrt{J_{A1}^2 + J_{A2}^2 - J_{A1}^2 J_{A2}^2}, K_{A1}K_{A2}\right)$$

4. Multiplication:

$$\tilde{A}_p \otimes \tilde{B}_p = \left(J_{A1}J_{A2}, \sqrt{K_{A1}^2 + K_{A2}^2 - K_{A1}^2 K_{A2}^2}\right)$$

5. Scalar multiplication:

$$\lambda\tilde{A}_p = \left(1 - (1 - J_{A1}^2)^T, K_A^T\right)$$

6. Complement:

$$\tilde{A}_p^C = (K_{A1}, J_{A1})$$

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2.8 Payoff:

Payoff is the quantity of satisfaction a player gets at the end of a play in a game.

2.9 Two persons zero sum game

A Game with two player I and Player II where the payoff to player II is the negative of the payoff to player I each day of the game so that the total sum of payoff is zero.

2.10 Matrix Game:

It is a two person zero sum game. It is defined as $\boldsymbol{m} \times \boldsymbol{n}$ matrix. $\boldsymbol{A} = [\boldsymbol{a_{ij}}]_{\boldsymbol{m \times n}}$ where i =1,2,3….m, j = 1,2,3…n a_{ij} are real numbers and A is called the pay off matrix. Hence the two players. Player I (row Player) has m choices and the player II (column player) has n choices available to them. If the player I choose to play choice i without knowing the choice of player II and player II choose to play choice j irrespective of the choice of player I, the payoff to player I Is a_{ij} and the payoff to player-II is - a_{ij}.

Both players want to choose strategies that will benefit the individual payoff. The payoff matrix A is given by

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{a_{11}} & \boldsymbol{a_{12}} \dots\dots\dots\dots & \boldsymbol{a_{1n}} \\ \boldsymbol{a_{21}} & \boldsymbol{a_{21}} \dots\dots\dots\dots & \boldsymbol{a_{2n}} \\ \boldsymbol{a_{m1}} & \boldsymbol{a_{m2}} \dots\dots\dots\dots & \boldsymbol{a_{mn}} \end{bmatrix}$$

2.11 Mixed strategy and pure strategy:

A mixed strategy of player I Is a probability distribution R over the rows of A, i.e., an element of the set $\boldsymbol{X} = \{\boldsymbol{x1}, \boldsymbol{x2}, \boldsymbol{x3} \dots \boldsymbol{xm}\} \in \boldsymbol{R^m}$, $\boldsymbol{x_i} \geq 0$, $\boldsymbol{x_i} = \sum_{i=1}^{m} \boldsymbol{1}$

Equivalently, a mixed strategy of player II is a probability distribution – Y over the column of A, i.e, an element of the set $\boldsymbol{y} = \{(\boldsymbol{y1}, \boldsymbol{y2}, \dots \boldsymbol{yn}) \in \boldsymbol{R^n}, \boldsymbol{y_j} \geq 0 \sum_{j=1}^{n} \boldsymbol{y_i} = \boldsymbol{1}\}$

A strategy $\boldsymbol{\lambda_i}$ of player I Is a pure strategy if does not involve an probability i.e., $\boldsymbol{\lambda_i}$ is an $\boldsymbol{x} \in \boldsymbol{x}$ with $\boldsymbol{x_i} = \boldsymbol{1}$ for forms I = 1,2,3,…m.

We denote the set of pure strategies of player I by $\boldsymbol{s_j} = \{\boldsymbol{\lambda 1}, \boldsymbol{\lambda 2}, \dots . \boldsymbol{\lambda m}\}$. A strategy Δ_i of player II is a pure strategy, if does not involve any probability, (i.e) Δ_i is an $\boldsymbol{y} \in \boldsymbol{Y}$ with $\boldsymbol{y_j} = \boldsymbol{1}$ for some $\boldsymbol{j} = \boldsymbol{1}, \boldsymbol{2}, \dots \boldsymbol{n}$

We denote the set of pure strategies of player II by $\boldsymbol{s} = \{\Delta_1, \Delta_2, \Delta_{3,\dots} \Delta_n\}$.

2.12 Maxmin and Minimax Principle:

The player I, who is the maximizing player lists the worst possible outcome of all his potential strategies, then he will choose that strategy, the most suitable for him which corresponds to one of these worst outcomes.

This is called maxi min principle of player I. In shortly, it is max_i, min_j $[a_{ij}]$. The player II who is the minimizing player lists the best possible outcomes of all potential strategies, then he will choose that strategy, the most suitable for him which correspond to one of these best outcomes. This is called minimum principle of player II. In shortly, it is min j max i $[a_{ij}]$.

2.13 Dominance Principle:

Dominance Principle is used to reduce the size of the payoff matrix by deleting those strategies which are dominated by others.

(i) If all the elements of a row say q^{th} row are ≤ the corresponding elements of any other row say r^{th} row, then the q^{th} row is dominated by r^{th} row.

(ii) If all the elements of a column say a^{th} column ≥ the corresponding elements of any other column, say the r^{th} column then q^{th} column is dominated by the r^{th} Column.

(iii) Dominated rows or columns are deleted and the optimal strategy will remain not affected.

2.14 Saddle Point:

For a matrix game $\boldsymbol{A} = [\boldsymbol{a_{ij}}]_{\boldsymbol{m \times n}}$, if $\max_{i} \min_{j} = \min_{j} \max_{i} [\boldsymbol{aij}]$, then the matrix game has saddle point at the (r,s) the position of A.

III. INNOVATIVE TOOL OF FERMATEAN FUZZY SET

3.1 Objectives:

To study and tackle the imprecision, vagueness and hesitancy involved to one of the decision-making problems say matrix games. In this article, we opt most and innovative tool, fermatean fuzzy set. The objective of this paper is to define the mathematical tool of fermatean fuzzy matrix game, study its properties and develop innovative algorithm to solve it. These innovative concepts are made lucid through numerical illustrations.

3.2 Methods:

The matrix games solved using fuzzy optimization tools and intuitonistic fuzzy optimization techniques are analysed. We came to the condition that the hesitancy associated with choosing strategies for players and pay off are not fully considered. Hence the strategy Pythagorean fuzzy optimization tool is used. The data set considered in this study are collected from a faculty of physical Education department in a college. Using the Innovative algorithm and data collected the study is completed

3.3 Findings:

The fermatean fuzzy expected pay off function and a new ranking function for fermatean fuzzy number is obtained in this paper. Moreover, the notion of fermatean fuzzy saddle point is also proposed. Then, we have introduced a novel algorithm called fermatean minimax dominian algorithm to solve fermatean fuzzy game problem and it is illustrated through numerical example.

3.4 Novelty:

This study explored concepts of game theory under fermatean fuzzy environment. The outlined mathematical

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model of fermatean fuzzy matrix game and the algorithm to Solve it is, novel in the research field.

3.5 Fermatean Furry payoff:

The Pay of a matrix game represented by fermatean fuzzy number (FFN) is called the fermatean fuzzy pay off. Due to uncertainty and imprecision in the game, ambiguity in the judgement of players, it is reliable to express pay off of a matrix game by FFN.

IV. MATHEMATICAL MODEL OF FERMATEAN FUZZY MATRIX GAME

Fermatean fuzzy matrix game is a two person zero sum game. It is defined by an

$\boldsymbol{m \times n}$ **fermatean fuzzy matrix** $\tilde{A} = [\tilde{a}_{ij}]_{m\times n}$ where $\boldsymbol{I \in \{1, 2, 3 \ldots\ldots m\}}$ are the m choices or strategies available to player I, $\boldsymbol{j \in \{1, 2, 3 \ldots\ldots m\}}$ are the n choices or strategies available to player II, $\tilde{a}_{ij}$ are FFN called the fermatean fuzzy payoff to player I for his jth choice against the jth choice of player.

The fermatean fuzzy pay off matrix is given by

$$\tilde{A} = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} \ldots\ldots\ldots\ldots. & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{21} \ldots\ldots\ldots\ldots. & \tilde{a}_{2n} \\ \tilde{a}_{m1} & \tilde{a}_{m2} \ldots\ldots\ldots\ldots. & \tilde{a}_{mn} \end{bmatrix}$$

The goal of the game is to find a strategy for player I that maximized fermatean fuzzy gain and find a strategy for player II that minimizes his fermatean fuzzy loss.

4.1 Fermatean fuzzy expected payoff Function:

Consider a fermatean fuzzy matrix game $\tilde{A} = [\tilde{a}_{ij}]_{m\times n}$ Let $\boldsymbol{x \in X}$ and $\boldsymbol{y \in Y}$ are mixed strategies of player I and II respectively. Then the frematean fuzzy expected payoff for player I Is

$$E(X,Y) = X^T AY$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{n} (x_i y_j \tilde{a}_{ij}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

It is also a fermatean fuzzy number. For the computation of equation (1) the arithmetic operations of fermatean fuzzy number are used.

4.2 Ranking Function:

A new ranking function defuzzification function will be exposed for fermatean fuzzy number (FFN). Let $F_{\sim}(R)$ denote the set of all fermatean fuzzy numbers. The ranking function maps $(F_{\tilde{A}_f})$ each fermatean fuzzy numbers to a real number.

It is defined as

$$F(\bar{A}_f) : \bar{F}(R) \to R$$

$$F(\bar{A}_f) = (J_A^2 + k_A^2)\left(\frac{1+\pi}{2}\right) \ldots\ldots\ldots\ldots\ldots \quad (1)$$

Where $\pi = \sqrt{1 - (J_A^2 + k_A^2)}$

Clearly $F_{\tilde{A}_f} \in [0, 1]$ Consider FFN $\tilde{A}_f$ and $\tilde{B}_f$

We can define a new order relation between $\tilde{A}_f$ and $\tilde{B}_f$ based on equation (1) as

1. $F(\bar{A}_f) \le p(\bar{B}_f) \Rightarrow \bar{A}_f \le \bar{B}_f$
2. $F(\bar{A}_f) \ge p(\bar{B}_f) \Rightarrow \bar{A}_f \ge \bar{B}_f$ ----------------------- (2)
3. $F(\bar{A}_f) = p(\bar{B}_f) \Rightarrow \bar{A}_f = \bar{B}_f$

The symbol ≤ denotes fermatean fuzzy inequality and has the linguistic interpretation as essentially less than or equal. Similarly for ≥ and ≅.

4.3 Fermatean Fuzzy Saddle Point

The point $X_0, Y_0, X_0 \in R^n$ is a fermatean Fuzzy saddle point if

$$E(X, X_0) \le \bar{E}(X_0, Y_0) \le \bar{E}(X_0, Y)\ \forall\ x \in R^n, \forall\ y \in R^n$$

4.4 Fermatean Minimax Dominian Algorithm

We propose a novel algorithm called fermatean minimax dominion algorithm to solve fermatean fuzzy matrix games.

It is developed on the basis of minimax principle and dominance principle of game theory combined with the proposed new ranking function of fermatean fuzzy number.

The steps of the algorithm are as follows:

Step 1: Consider the fermatean fuzzy payoff matrix of the game whose payoff are expressed by FFN.

Step 2: Defuzzifying the fermatean fuzzy payoff using the newly proposed ranking function of FFN given (1)

Step 3: Check for fermatean fuzzy saddle point using minimax and maximum principle. If exist go to step 4, otherwise step 5.

Step 4: Determine the fermatean fuzzy saddle point, optimal strategies of player I and II and frematean fuzzy value of game.

Step 5: Apply Dominance principle to the row and columns of defuzzified matrix obtained in step 2 and reduce it to a $\boldsymbol{2 \times 2}$ matrix.

Step 6: Determine the Optimal matrix strategies of player I and player II.

Step 7 : Determine the fermatean fuzzy value of the game by using V= $\frac{a_{11}a_{22} - a_{12}a_{21}}{\sqrt{a_{13}a_{23} + a_{31}a_{21}}}$

V. NUMERICAL EXAMPLE

Example 1:

The best way to demonstrate the proposed fermatean minimize dominian algorithm to solve fermatean fuzzy matrix game is by virtue of an example. Here we will take in to account two numerical examples, one with fermatean fuzzy saddle point and the other without fermatean fuzzy saddle point for the better comprehension of the theory.

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Step-1: Consider a fermatean fuzzy matrix

$$\boldsymbol{P} = \begin{vmatrix} [0.6, 0.7] & [0.7, 0.3] & [0.3, 0.2] \\ [0.3, 0.9] & [0.5, 0.6] & [0.4, 0.8] \\ [0.8, 0.4] & [0.4, 0.7] & [0.5, 0.7] \end{vmatrix}$$

By defuzzification (Apply -2) with the help of

$$\boldsymbol{P} = (J_A^2 + k_A^2)\left(\frac{1+\pi}{2}\right) \text{ Where } \boldsymbol{\pi} = \sqrt{1 - (J_A^2 + k_A^2)}$$

We have

$$\boldsymbol{P} = \begin{bmatrix} 0.5895 & 0.4779 & 0.1256 \\ 0.5922 & 0.4954 & 0.1624 \\ 0.5788 & 0.5172 & 0.5586 \end{bmatrix}$$

The value of Row = { $\mathbf{0.1256, 0.1624, 0.5172}$ }

Max {Row Minimum} = $\mathbf{0.5172}$

Max value of Column = { $\mathbf{0.5922, 0.5172, 0.5586}$ }

Max – value of Column = {0.5922, 0.5172, 0.5586}

Min {Column Maxima} =0.5172

Max {Row Minimum} = Min {Column Maximum}

Hence by Minimax and Maxmin principle, saddle point exists at (3,2)th position.

Optimum Pure strategy player I= (0,0,1)

Optimum Pure strategy player II= (0,1,0)

In this problem saddle point exists.

Suppose the saddle point does not exists we have to follow the following set of procedure to optimize the value of Game.

Example 2: The best way to demonstrate the proposed fermatean minimize dominion algorithm to solve fermatean fuzzy matrix game is by virtue of an example. Here we will take in to account two numerical examples, one with fermatean fuzzy saddle point and the other without fermatean fuzzy saddle point for the better comprehension of the theory.

Step-1: Consider a fermatean fuzzy matrix

$$\boldsymbol{P} = \begin{vmatrix} [0.6, 0.7] & [0.7, 0.3] & [0.4, 0.6] & [0.3, 0.2] \\ [0.3, 0.9] & [0.5, 0.6] & [0.7, 0.4] & [0.4, 0.1] \\ [0.8, 0.4] & [0.4, 0.7] & [0.7, 0.7] & [0.5, 0.7] \\ [0.4, 0.6] & [0.5, 0.7] & [0.8, 0.4] & [0.6, 0.7] \end{vmatrix}$$

Apply step 2

By defuzzification with the help of

$$\boldsymbol{P} = (J_A^2 + k_A^2)\left(\frac{1+\pi}{2}\right) \text{ Where } \boldsymbol{\pi} = \sqrt{1 - (J_A^2 + k_A^2)}$$

We have

$$= \begin{vmatrix} 0.5895 & 0.4779 & 0.4401 & 0.1256 \\ 0.5922 & 0.4954 & 0.5172 & 0.1624 \\ 0.5788 & 0.5172 & 0.5895 & 0.5586 \\ 0.4401 & 0.5586 & 0.4401 & 0.5895 \end{vmatrix}$$

Using step – 3

Min Row values = { $\mathbf{0.1256, 0.1624, 0.5172, 0.4401}$ }

Therefore Maximum {Row Minimum} = $\mathbf{0.5172}$

Max Column Value = { $\mathbf{0.5922, 0.5586, 0.5895, 0.5895}$ }

Therefore {Column Max}= $\mathbf{0.5586}$

Therefore Max {Row Min} ≠Minimum {Column Max}

Hence saddle point does not exist.

Apply steps 5 to the Rows and Columns.

If the above matrix, we can get

$$\boldsymbol{P} = \begin{vmatrix} 0.5895 & 0.4401 & 0.1256 \\ 0.5788 & 0.5895 & 0.5586 \\ 0.4401 & 0.4401 & 0.5895 \end{vmatrix}$$

Value of the game is obtained by

$$\boldsymbol{V} = \frac{a_{11}a_{22} - a_{12}a_{21}}{\sqrt{a_{13}a_{23} + a_{31}a_{21}}}$$

$$\boldsymbol{V} = \frac{(0.5895 \times 0.5895) - (0.1256 \times 0.4401)}{\sqrt{(0.1256 \times 0.5586) + (0.4401 \times 0.5788)}}$$

$$\boldsymbol{V} = \mathbf{0.57}$$

VI. CONCLUSION

The fermatean fuzzy expected pay off function and a new ranking function for fermatean fuzzy number is obtained in this paper. Moreover, the notion of fermatean fuzzy saddle point is also proposed. Then, we have introduced a novel algorithm called fermatean minimax dominion algorithm to solve fermatean fuzzy game problem and it is illustrated through numerical example. This study explored concepts of game theory under fermatean fuzzy environment. The outlined mathematical model of fermatean fuzzy matrix game and the algorithm to solve it is, novel in the research field.

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